1. **In this question you should show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

Using algebra, solve the inequality

$$x^2 - x > 20$$

writing your answer in set notation.

(3)

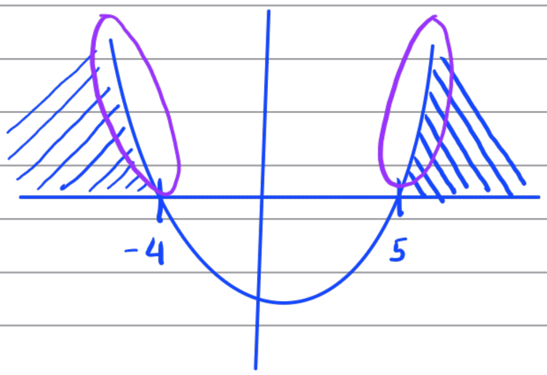
$x^2 - x > 20$

$= x^2 - x - 20 > 0$ — Put the value into calculator

$= (x-5)(x+4) > 0$

$x > 5$ and $x < -4$ (1)

Critical values $x = 5$ and $x = -4$



(1)

inequality satisfied when $x < -4$ and $x > 5$

In set notation :

$\{x : x < -4\} \cup \{x : x > 5\}$ (1)

2.

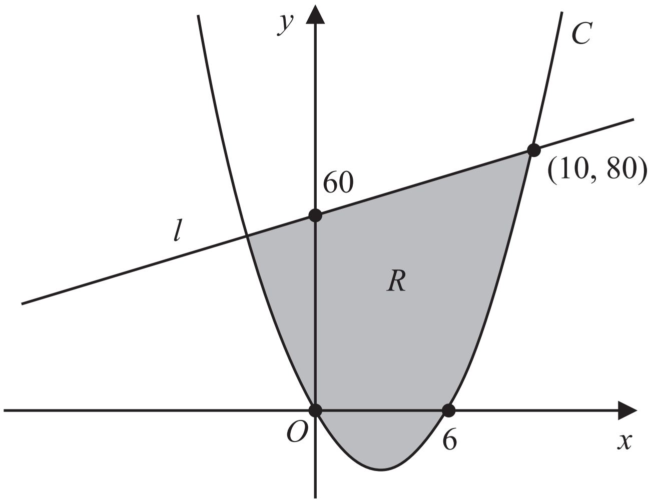


Figure 3

Figure 3 shows a sketch of a curve C and a straight line l.

Given that

- C has equation $y = \mathrm{f}(x)$ where $\mathrm{f}(x)$ is a quadratic expression in x
- C cuts the x-axis at 0 and 6
- l cuts the y-axis at 60 and intersects C at the point $(10, 80)$

use inequalities to define the region R shown shaded in Figure 3.

(5)

has all values needed to form equation

Finding the equation of line l :

gradient of l : $\frac{80-60}{10} = 2$ 1

y-intercept : 60

Hence, $y = 2x + 60$ 1 — equation of l

Finding equation of curve C :

$y = ax^2 + bx$

at $(6,0)$: $0 = 36a + 6b$ — ①

at $(10,80)$: $80 = 100a + 10b$ — ② (1)

from ①

$b = \frac{-36a}{6} = -6a$ — ③

Substitute ③ into ②

$80 = 100a + 10(-6a)$

$80 = 40a$

$a = 2$, $b = -12$

$y = 2x^2 - 12x$

$= 2x(x-6)$ (1) — equation of curve C

Region R : $2x(x-6) \leq y \leq 2x + 60$ (1)

3.

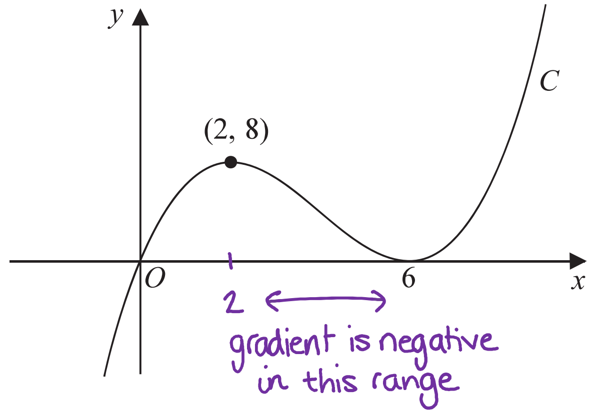


Figure 1

Figure 1 shows a sketch of a curve C with equation $y = \mathrm{f}(x)$ where $\mathrm{f}(x)$ is a cubic expression in x.

The curve

- passes through the origin
- has a maximum turning point at $(2, 8)$
- has a minimum turning point at $(6, 0)$

(a) Write down the set of values of x for which

$$\mathrm{f}'(x) < 0$$

(1)

The line with equation $y = k$, where k is a constant, intersects C at only one point.

(b) Find the set of values of k, giving your answer in set notation.

(2)

(c) Find the equation of C. You may leave your answer in factorised form.

(3)

(a) $2 < x < 6$ 1

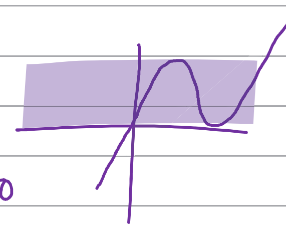
$f'(x) < 0$ means the gradient is negative. Negative gradient = line going down. ↘

(b) $k > 8$ or $k < 0$ 1

y=k is a horizontal line through the y-axis.

$\{k : k > 8\} \cup \{k : k < 0\}$ 1

has to be outside of shaded region to intersect only once. →



CHOOSE ONE OF THESE METHODS.

(c) Method 1 : Recognise curve has form $y = ax(x-6)^2$ ① states form of c

$(2,8) \rightarrow \quad 8 = 2a(2-6)^2$ ①

$8 = 32a$

$a = \frac{1}{4}$

$\therefore y = \frac{1}{4}x(x-6)^2$ ①

Method 2 : Solving Simultaneous Equations

$y = ax^3 + bx^2 + cx \quad \leftarrow$ no +d because the curve goes through the origin.

when $x = 2$, $y = 8$:

$8 = a(2^3) + b(2^2) + c(2)$

① $4 = 4a + 2b + c$

when $x = 6$, $y = 0$:

$0 = a(6^3) + b(6^2) + c(6)$

② $0 = 216a + 36b + 6c$ ① for 2 sim. eq.

$f'(x) = 3ax^2 + 2bx + c$

when $x = 6$, $f'(x) = 0$: $\leftarrow$ $(6,0)$ is a turning point

$0 = 3a(6^2) + 2b(6) + c$

③ $0 = 108a + 12b + c$

Solve ①, ②, ③ simultaneously : $\leftarrow$ use a calculator or solve by hand.

$4 = 4a + 2b + c$

$0 = 216a + 36b + 6c$

$0 = 108a + 12b + c$

$a = \frac{1}{4}$, $b = -3$, $c = 9$ ① for solving sim. eq.

$y = \frac{1}{4}x^3 - 3x^2 + 9x$ ①

4. (i) Given that p and q are integers such that

$$pq \text{ is even}$$

use algebra to prove by contradiction that at least one of p or q is even.

(3)

(ii) Given that x and y are integers such that

- $x < 0$
- $(x + y)^2 < 9x^2 + y^2$

show that $y > 4x$

(2)

(i) There exists integers p and q such that pq is even and p and q are both odd. 1

write out the contradiction.

Let $p = 2m+1$ and $q = 2n+1$

we know that $2 \times$ any number is even by definition, therefore $2m+1$ is odd by definition.

$pq = (2m+1)(2n+1)$
$= 4mn + 2n + 2m + 1$ 1
$= 2(2mn + n + m) + 1$

1 This is of the form $2a+1$, so is odd.

This is a contradiction, therefore if pq is even, then at least one of p and q must be even.

(ii) $(x+y)^2 < 9x^2 + y^2$
$x^2 + 2xy + y^2 < 9x^2 + y^2$ $\quad -y^2, -x^2$
$2xy < 8x^2$ 1 $\quad \div x$
$2y < 8x$ $\quad -8x$
$2y - 8x < 0$

$x < 0$ so $8x < 0$. $2y - 8x > 0$, so $2y > 8x$.

$2y > 8x$ $\quad \div 2$
$y > 4x$ 1